hep-th/9707081

Large-N limit of the generalized 2-dimensional Yang-Mills theories

M. Alimohammadi[a,b,1], **M. Khorrami**[a,b,c,2], **A. Aghamohammadi**[b,d3]

[a] *Department of Physics, Tehran University, North-Kargar Ave. Tehran, Iran*
[b] *Institute for Studies in Theoretical Physics and Mathematics, P.O.Box 5531, Tehran 19395, Iran*
[c] *Institute for Advanced Studies in Basic Sciences, P.O.Box 159, Gava Zang, Zanjan 45195, Iran*
[d] *Department of Physics, Alzahra University, Tehran 19834, Iran*

Abstract

Using the standard saddle-point method, we find an explicit relation for the large-N limit of the free energy of an arbitrary generalized 2D Yang-Mills theory in the weak ($A < A_c$) region. In the strong ($A > A_c$) region, we investigate carefully the specific fourth Casimir theory, and show that the ordinary integral equation of the density function is not adequate to find the solution. There exist, however, another equation which restricts the parameters. So one can find the free energy in strong region and show that the theory has a third order phase transition.

[1] e-mail:alimohmd@netware2.ipm.ac.ir
[2] e-mail:mamwad@netware2.ipm.ac.ir
[3] e-mail:mohamadi@netware2.ipm.ac.ir

1 Introduction

The pure 2D Yang-Mills ($\mathrm{tr}(F^2)$) theory (YM_2) defined on compact Riemann surfaces, is characterized by its invariance under area preserving diffeomorphism and the fact that there are no propagating degrees of freedom. This theory is not unique in this sense, and it is possible to generalize it without losing these properties.

In an equivalent formulation of YM_2 theory, one can use $i\mathrm{tr}(BF) + \mathrm{tr}(B^2)$ as the Lagrangian, where B is an auxiliary pseudo-scalar field in the adjoint representation of the gauge group. Path integration over the field B leaves an effective Lagrangian of the form $\mathrm{tr}(F^2)$ [1]. Now, a generalized 2D Yang-Mills theory (gYM_2) is a theory with above Lagrangian, in which the term $\mathrm{tr}(B^2)$ is replaced by an arbitrary class function $\Lambda(B)$.

In [2], where these theories were first introduced, the partition function of gYM_2 have been obtained by considering its action as a perturbation of the topological theory at zero area. In [3], these theories have been coupled to fermions, to obtain the generalized QCD_2. Generalizing the Migdal's suggestion about the local factor of plaquettes [4], the authors of [5] have found the partition function and the expectation values of the Wilson loops for gYM_2's. In [6], these quantities and also the generating functional of the field strength have been calculated for arbitrary two dimensional orientable and nonorientable surfaces, using the standard path integral method.

One of the important features of YM_2, and also gYM_2's, is their behaviour in the case of large gauge groups, e.g., the large N behaviour of $SU(N)$ (or $U(N)$) gauge theories. On one hand, it is interesting to study the relation between these theories at large N and the string theory. Such investigations began in [7] and [8] for YM_2 and in [5] for gYM_2. In these papers, it was shown that the coefficients of $1/N$ expansion of the partition function of $SU(N)$ gauge theories are determined by a sum over maps from a two-dimensional surface onto the two-dimensional target space.

Another thing of interest, also related to the above mentioned one, is to study the large N behaviour of the free energy of these theories. This is done by replacing the sum over the irreducible representations of $SU(N)$ (or $U(N)$), appearing in the expressions of partition function and Green functions, by a path integral over continuous Young tableaux, and calculating the area-dependence of the physical quantities from the saddle-point configuration. In [9], It was shown that the free energy of $U(N)$ YM_2 on a sphere with surface area $A < A_c = \pi^2$ has a logarithmic behaviour. Later, the authors of [10] calculated the free energy for surface area $A > \pi^2$, and showed that the YM_2 has a third order phase transition at the critical surface area. Such kind of calculation for gYM_2 began in [11], and the authors claimed that these theories have a rich phase structure. They also discussed the general behaviour of the density function ρ for $A < A_c$, and brought some comments for the $A > A_c$ region.

In this paper, we first complete the calculation of the free energy of $U(N)$ gYM$_2$ for $A < A_c$ region for an arbitrary gYM$_2$ on sphere and derive an exact solution for the density function. But the main part of the paper is about the $A > A_c$ region of a specific fourth Casimir model. It is seen that here, unlike the YM$_2$ model, the density function ρ has two maxima, for $A < A_c$. So some new features arise in the phase transition, which must be handled more carefully. At the end we show, however, that for this particular model there exist a third order phase transition, just as in the case of YM$_2$. This is another similarity between these two theories.

2 Large-N behaviour of gYM$_2$ at $A < A_c$

The partition function of the gYM$_2$ on a sphere is [5,6]

$$Z = \sum_r d_r^2 e^{-A\Lambda(r)}, \tag{1}$$

where r's label the irreducible representations of the gauge group, d_r is the dimension of the r'th representation, A is the area of the sphere and $\Lambda(r)$ is :

$$\Lambda(r) = \sum_{k=1}^{p} \frac{a_k}{N^{k-1}} C_k(r), \tag{2}$$

in which C_k is the k'th Casimir of group, and a_k's are arbitrary constants. Now consider the gauge group $U(N)$ and parametrize its representation by $n_1 \geq n_2 \geq \cdots \geq n_N$, where n_i is the length of the i'th row of the Young tableau. It is found that [12]

$$d_r = \prod_{1\leq i<j\leq N} \left(1 + \frac{n_i - n_j}{j-i}\right)$$

$$C_k = \sum_{i=1}^{N} [(n_i + N - i)^k - (N-i)^k]. \tag{3}$$

To make the partition function (1) convergent, it is necessary that p in eq.(2) be even and $a_p > 0$.

Now, following [9], we can write the partition function (1) at large N, as a path integral over continuous parameters. We introduce the continuous function :

$$\phi(x) = -n(x) - 1 + x, \tag{4}$$

where

$$0 \leq x := i/N \leq 1 \quad \text{and} \quad n(x) := n_i/N. \tag{5}$$

The partition function (1) then becomes

$$Z = \int \prod_{0\leq x\leq 1} d\phi(x) e^{S[\phi(x)]}, \tag{6}$$

where

$$S(\phi) = N^2\{-A\int_0^1 dxG[\phi(x)] + \int_0^1 dx\int_0^1 dy \ \ \log|\phi(x)-\phi(y)| \ \ \}, \tag{7}$$

apart from an unimportant constant, and :

$$G(\phi) = \sum_{k=1}^{p}(-1)^k a_k \phi^k. \tag{8}$$

As $N \to \infty$, the partition function (7) is determined by the configuration which maximizes S. The saddle point equation for S is :

$$g[\phi(x)] = \mathrm{P}\int_0^1 \frac{dy}{\phi(x)-\phi(y)}, \tag{9}$$

where:

$$g(\phi) = \frac{A}{2}G'(\phi), \tag{10}$$

and P indicates the principal value of the integral. Introducing the density

$$\rho[\phi(x)] = \frac{dx}{d\phi(x)}, \tag{11}$$

the eq.(9) is reduced to:

$$g(z) = \mathrm{P}\int_b^a \frac{\rho(\lambda)d\lambda}{z-\lambda}, \tag{12}$$

along with the normalization condition

$$\int_b^a \rho(\lambda)d\lambda = 1. \tag{13}$$

The condition $n_1 \geq n_2 \geq \cdots \geq n_N$ imposes the following condition on the density $\rho(\lambda)$:

$$\rho(\lambda) \leq 1. \tag{14}$$

To solve eq.(12), we define the function $H(z)$ in complex z-plane [13] ,

$$H(z) := \int_b^a \frac{\rho(\lambda)d\lambda}{z-\lambda}. \tag{15}$$

This function is analytic on the complex plane exept for a cut at $[b,a]$. There , one has

$$H(z \pm i\epsilon) = g(z) \mp i\pi\rho(z) \qquad b \leq z \leq a. \tag{16}$$

H is found to be [10,14] :

$$H(z) = \frac{1}{2\pi i}\sqrt{(z-a)(z-b)}\oint_c \frac{g(\lambda)d\lambda}{(z-\lambda)\sqrt{(\lambda-a)(\lambda-b)}}, \tag{17}$$

where c is a contour encircling the cut $[b, a]$, and excluding z and $g(\lambda)$ is defined through eq.(10). Deforming c to a contour around the point z and the contour c_∞ (a contour at the infinity), one finds:

$$H(z) = g(z) - \sqrt{(z-a)(z-b)} \sum_{m,n,q=0}^{\infty} \frac{(2n-1)!!(2q-1)!!}{2^{n+q} n! q! (n+q+m+1)!} a^n b^q z^m g^{(n+q+m+1)}(0), \quad (18)$$

where $g^{(n)}$ is the n-th derivative of g. It follows from (15) and (13) that $H(z)$ behaves like $1/z$ at $z \to \infty$, or $(z-a)^{-1/2}(z-b)^{-1/2}H(z)$ behaves like $1/z^2$ at $z \to \infty$. Using this, one can expand $(z-a)^{-1/2}(z-b)^{-1/2}H(z)$, equate the coefficients of $1/z$ and $1/z^2$ equal to 0 and 1, respectively, and arrive at

$$\sum_{n,q=0}^{\infty} \frac{(2n-1)!!(2q-1)!!}{2^{n+q} n! q! (n+q)!} a^n b^q g^{(n+q)}(0) = 0, \quad (19)$$

and

$$\sum_{n,q=0}^{\infty} \frac{(2n-1)!!(2q-1)!!}{2^{n+q} n! q! (n+q-1)!} a^n b^q g^{(n+q-1)}(0) = 1. \quad (20)$$

These equations determine a and b. From (16), ρ is determined :

$$\rho(z) = \frac{\sqrt{(a-z)(z-b)}}{\pi} \sum_{m,n,q=0}^{\infty} \frac{(2n-1)!!(2q-1)!!}{2^{n+q} n! q! (m+n+q+1)!} a^n b^q z^m g^{(m+n+q+1)}(0). \quad (21)$$

Defining the free energy as

$$F := -\frac{1}{N^2} \ln Z, \quad (22)$$

one has

$$F'(A) = \int_0^1 dx \;\; G[\phi(x)] = \int_b^a d\lambda \;\; G(\lambda)\rho(\lambda). \quad (23)$$

Expanding $H(z)$ for $z \to \infty$, one can find the integrals

$$\int_b^a d\lambda \;\; \lambda^n \rho(\lambda), \quad (24)$$

by using the eqs.(15) and (18). Knowing these, one can calculate $F'(A)$ through (23).

It is also worth mentioning that if G is an even function, then $b = -a$, ρ is an even function, and equations (17), (18), (20) and (21) become

$$H(z) = \frac{1}{2\pi i} \sqrt{z^2 - a^2} \oint_c \frac{g(\lambda) d\lambda}{(z-\lambda)\sqrt{\lambda^2 - a^2}}, \quad (17)'$$

$$H(z) = g(z) - \sqrt{z^2 - a^2} \sum_{n,q=0}^{\infty} \frac{(2n-1)!!}{2^n n! (2n+q+1)!} a^{2n} z^q g^{(2n+q+1)}(0), \quad (18)'$$

$$\sum_{n=0}^{\infty} \frac{(2n-1)!!}{2^n n! (2n-1)!} a^{2n} g^{(2n-1)}(0) = 1, \quad (20)'$$

and

$$\rho(z) = \frac{\sqrt{a^2-z^2}}{\pi}\sum_{n,q=0}^{\infty}\frac{(2n-1)!!}{2^n n!(2n+q+1)!}a^{2n}z^q g^{(2n+q+1)}(0). \tag{21$'$}$$

As the simplest example of gYM$_2$, consider $G(\phi)=\phi^4$. Using the above relations, one finds

$$\begin{aligned}\rho(z) &= \frac{2A}{\pi}(\frac{a^2}{2}+z^2)\sqrt{a^2-z^2},\\ a &= (\frac{4}{3A})^{1/4},\\ F'(A) &= \frac{1}{4A}.\end{aligned} \tag{25}$$

The density ρ has a minimum at $z=0$ and two maxima at $z_0^{(\pm)}=\pm a/\sqrt{2}$. Now as $\rho(z_0^{(\pm)})=\sqrt{2}a^3A/\pi$, if $A>A_c=27\pi^4/256$, then the condition $\rho\leq 1$ is violated. So the solution (25) is valid only in the region $A\leq A_c$.

It is worth noting that the function $F'(A)$ for $G(\phi)=\phi^k$ can be simply found by rescaling the field ϕ by $A^{1/k}$ in eq.(7) (as noted in [10]) , after which the only A-dependent term of S is $\log A^{1/k}$ and therefore $F'(A)=\frac{1}{kA}$.

3 The $G(\phi)=\phi^4$ model in the region $A>A_c$

In this region, the solution (25) is not correct any more. As $\rho(z)$ for $A<A_c$ has two symmetric maxima, we use the following ansatz for ρ :

$$\rho_s(z)=\begin{cases}1, & z\in[-b,-c]\bigcup[c,b]=:L'\\ \tilde{\rho}_s(z), & z\in[-a,-b]\bigcup[-c,c]\bigcup[b,a]=:L.\end{cases} \tag{26}$$

Putting this in (7), we have

$$S(\rho_s)=N^2\Big[\ -A\int_{-a}^{a}dz\ \ \rho_s(z)z^4+\int_{-a}^{a}dz\int_{-a}^{a}dw\rho_s(z)\rho_s(w)\log|z-w|\ \Big]. \tag{27}$$

To maximize this along with the condition (13), we introduce the Lagrange multiplier μ and the functional $\tilde{S}$:

$$\tilde{S}=S+N^2\mu\Big[\ \int_{-a}^{a}dz\ \ \rho_s(z)-1\ \Big]. \tag{28}$$

The variation of $\tilde{S}$ is

$$\delta\tilde{S}=N^2\int_{-a}^{a}dz\Big[\ -Az^4+2\int_{-a}^{a}dw\ \ \rho_s(w)\log|z-w|+\mu\ \Big]\delta\rho_s+N^2\Big[\ \int_{-a}^{a}dz\ \ \rho_s(z)-1\ \Big]\delta\mu, \tag{29}$$

where

$$\delta\rho_s(z)=\begin{cases}0, & z\in L'\\ \delta\tilde{\rho}_s(z), & z\in L.\end{cases} \tag{30}$$

Equating $\delta\tilde{S}$ to zero, we have

$$\frac{A}{2}z^4 - \int_{-a}^{a} dw \quad \rho_s(w)\log|z-w| = \frac{\mu}{2}, \qquad z \in L, \tag{31}$$

and

$$\int_{-a}^{a} dz \quad \rho_s(z) = 1. \tag{32}$$

Differentiating (31) with respect to z, we have

$$2Az^3 = \mathrm{P}\int_{-a}^{a} dw \quad \frac{\rho_s(w)}{z-w}, \qquad z \in L. \tag{33}$$

This equation is, however, not equivalent to (31), since L is a disconnected set. In fact, (31) is equivalent to (33) and the following equation

$$\frac{A}{2}(b^4 - c^4) - \int_{-a}^{a} dw \quad \rho_s(w)\log|\frac{b-w}{c-w}| = 0, \tag{34}$$

which is difference of eq.(31) at $z=b$ and $z=c$. The above equation can also be written as

$$\int_{c}^{b} dz \left[\quad 2Az^3 - \mathrm{P}\int_{-a}^{a} dw \quad \frac{\rho_s(w)}{z-w} \right] = 0. \tag{35}$$

So our task is to solve the eq.(33), along with the conditions (32) and (35). To do so, we use the procedure of the previous section:

$$\begin{aligned} H_s(z) := \mathrm{P}\int_{-a}^{a} dw \quad \frac{\rho_s(w)}{z-w} &= \int_{L} dw \quad \frac{\tilde{\rho}_s(w)}{z-w} + \log\frac{z+b}{z+c} + \log\frac{z-c}{z-b} \\ &:= \tilde{H}_s(z) + \log\frac{z+b}{z+c} + \log\frac{z-c}{z-b} \end{aligned} \tag{36}$$

where $\tilde{H}_s$ has a three cut singularity at $z \in L$. Similar to eq. (17), the solution of $\tilde{H}_s$ is [14] :

$$\tilde{H}_s(z) = \frac{1}{2\pi i}\sqrt{(z^2-a^2)(z^2-b^2)(z^2-c^2)} \oint_{c_L} \frac{\tilde{g}(\lambda)d\lambda}{(z-\lambda)\sqrt{(\lambda^2-a^2)(\lambda^2-b^2)(\lambda^2-c^2)}}, \tag{37}$$

where

$$\begin{aligned} \tilde{g}(z) &:= g(z) - \log\frac{z+b}{z+c} - \log\frac{z-c}{z-b} \\ &= 2Az^3 - \log\frac{z+b}{z+c} - \log\frac{z-c}{z-b}, \end{aligned} \tag{38}$$

and c_L is a contour encircling the three distinct intervals of L. Deforming c_L to a contour at infinity and three contours encircling the point z and the two intervals $[-b,-c]$ and $[c,b]$, respectively, one can determine $\tilde{H}_s$ to be

$$\tilde{H}_s(z) = \tilde{g}(z) - 2\sqrt{(z^2-a^2)(z^2-b^2)(z^2-c^2)} \left(A + \int_{c}^{b} \frac{\lambda d\lambda}{(z^2-\lambda^2)\sqrt{(a^2-\lambda^2)(b^2-\lambda^2)(\lambda^2-c^2)}} \right), \tag{39}$$

and

$$H_s(z) = 2Az^3 - 2\sqrt{(z^2-a^2)(z^2-b^2)(z^2-c^2)}\left(A + \int_c^b \frac{\lambda d\lambda}{(z^2-\lambda^2)\sqrt{(a^2-\lambda^2)(b^2-\lambda^2)(\lambda^2-c^2)}}\right). \tag{40}$$

Using the definition of $H_s(z)$ in (36), we arrive at the following expansion for $H_s(z)$ at large z

$$H_s(z) = \frac{1}{z} + \frac{1}{z^3}\int_{-a}^{a} \rho_s(\lambda)\lambda^2 d\lambda + \frac{1}{z^5}F'_s(A) + \cdots, \tag{41}$$

where F_s is the free energy in the strong region. Therefore, one can expand $H_s(z)/\sqrt{(z^2-a^2)(z^2-b^2)(z^2-c^2)}$ and demand that it behaves like $1/z^4$ at large z. This gives

$$A(a^2+b^2+c^2) = 2\int_c^b \frac{\lambda d\lambda}{\sqrt{(a^2-\lambda^2)(b^2-\lambda^2)(\lambda^2-c^2)}}, \tag{42}$$

and

$$A\left[\ \frac{3}{4}(a^4+b^4+c^4) + \frac{1}{2}(a^2b^2+b^2c^2+c^2a^2)\ \right] - 2\int_c^b \frac{\lambda^3 d\lambda}{\sqrt{(a^2-\lambda^2)(b^2-\lambda^2)(\lambda^2-c^2)}} = 1. \tag{43}$$

Finally, the $1/z^5$ coefficient of $H_s(z)$ is $F'_s(A)$. So,

$$\begin{aligned} F'_s(A) = \frac{A}{16}\Big[\ &\frac{5}{4}(a^8+b^8+c^8) - \frac{1}{2}(a^4b^4+a^4c^4+b^4c^4) + (a^2b^2c^4+a^2c^2b^4+b^2c^2a^4) \\ &-(a^2b^6+a^2c^6+b^2a^6+b^2c^6+c^2a^6+c^2b^6)\ \Big] \\ +\frac{1}{8}\Big[\ &a^6+b^6+c^6-(a^2b^4+a^2c^4+b^2a^4+b^2c^4+c^2a^4+c^2b^4)+2a^2b^2c^2\ \Big]\int_c^b \frac{\lambda d\lambda}{R(\lambda)} \\ +\frac{1}{4}\Big[\ &a^4+b^4+c^4-2(a^2b^2+a^2c^2+b^2c^2)\ \Big]\int_c^b \frac{\lambda^3 d\lambda}{R(\lambda)} + (a^2+b^2+c^2)\int_c^b \frac{\lambda^5 d\lambda}{R(\lambda)} - 2\int_c^b \frac{\lambda^7 d\lambda}{R(\lambda)}, \end{aligned} \tag{44}$$

where

$$R(\lambda) := \sqrt{(a^2-\lambda^2)(b^2-\lambda^2)(\lambda^2-c^2)}. \tag{45}$$

Note that (42) and (43) are just two equations for three unknowns a, b and c. This is the same situation encountered in [15]. There, the authors have considered an asymmetric ansatz for the density of YM_2, and have found three equations for four parameters. We will discuss this problem in [16]. Now, for the case in hand, the third equation is (35), which using eqs. (36) and (37) can be rewritten as

$$A\int_c^b R(z)dz + \int_c^b dz\ \ \mathrm{P}\int_c^b \ \frac{R(z)\lambda d\lambda}{(z^2-\lambda^2)R(\lambda)} = 0. \tag{46}$$

Now we have three equations ((42),(43), and (46)) for three unknowm parameters a, b and c.

To study the structure of the phase transition, we use the following change of variables

$$c = s(1-y),$$
$$b = s(1+y), \tag{47}$$
$$a = s\sqrt{2+e}.$$

Here, e and y are equal to zero at the transition point ($A = A_c$). Now, one can expand the equations (42), (43) and (46) and obtain

$$4As^2 - \frac{\pi}{s} + (As^2 + \frac{\pi}{2s})e - \frac{3\pi e^2}{8s} + (2As^2 - \frac{5\pi}{4s})y^2 + \frac{21\pi e y^2}{8s} - \frac{249}{64}\frac{\pi y^4}{s} = 0, \tag{48}$$

$$7As^4 - 1 - \pi s + (4As^4 + \frac{\pi s}{2})e + (\frac{3As^4}{4} - \frac{3\pi s}{8})e^2 + (10As^4 - \frac{13\pi s}{4})y^2$$
$$+ (As^4 + \frac{37\pi s}{8})ey^2 + (2As^4 - \frac{545}{64}\pi s)y^4 = 0, \tag{49}$$

and

$$2\pi - 8As^3 - 4Aes^3 - 2\pi e + Ae^2 s^3 + 2\pi e^2 + \frac{1}{4}(16As^3 + 17\pi - 8Aes^3 + 46\pi e)y^2$$
$$+ (2As^3 + \frac{491}{32}\pi)y^4 = 0, \tag{50}$$

respectively. These expansions are up to order y^4, or e^2 (as we will see e is of the order y^2). Solving eq. (48) for s results

$$s = (\frac{\pi}{4A})^{1/3}\left(1 - \frac{e}{4} + \frac{e^2}{8} + \frac{y^2}{4} - \frac{11ey^2}{16} + \frac{71y^4}{64}\right), \tag{51}$$

from which, one obtains (using eq.(50))

$$e = \frac{5}{2}y^2 - \frac{3}{16}y^4. \tag{52}$$

This shows that e is, in fact, of the order y^2. Substituting these into the eq.(49), we arrive at

$$y^2 = \frac{\alpha}{3} + \frac{7}{96}\alpha^2, \tag{53}$$

and

$$e = \frac{5}{6}\alpha + \frac{31}{192}\alpha^2, \tag{54}$$

where α is the reduced area, $\alpha = \frac{A-A_c}{A_c}$. Now $F_s'(A)$ in eq. (44) can be calculated. The result is

$$F_s'(A) = \frac{1}{4A}(1 + \frac{4}{27}\alpha^2 + \cdots). \tag{55}$$

Comparing this with $F'_w(A)$, eq.(25), it is seen that

$$F'_s(A) - F'_w(A) = \frac{1}{27A_c}(\frac{A - A_c}{A_c})^2 + \cdots. \tag{56}$$

This shows that we have a third order phase transition, which is the same as the ordinary YM_2 [10].

Acknowledgement M. Alimohammadi would like to thank the research vice-chancellor of Tehran University, this work was partially supported by them.

References

[1] M. Blau and G. Thompson, "Lectures on $2d$ Gauge Theories", Proceedings of the 1993 Trieste Summer School on High Energy Physics and Cosmology, World Scientific, Singapore (1994) 175-244.

[2] E. Witten, J. Geom. Phys. **9** (1992) 303.

[3] M. R. Douglas, K. Li and M. Staudacher, Nucl. Phys. **B240** (1994) 140.

[4] A. Migdal, Zh. Eskp. Teor. Fiz. **69** (1975) 810 (Sov. Phys. JETP **42**, 413).

[5] O. Ganor, J. Sonnenschein, and S. Yankielowicz, Nucl. Phys. **B434** (1995) 139.

[6] M. Khorrami and M. Alimohammadi, Mod. Phys. Lett. **A12** (1997) 2265.

[7] D. J. Gross, Nucl. Phys. **B400** (1993) 161.

[8] D. J. Gross and W. Taylor, Nucl. Phys. **B400** (1993) 181.

[9] B. Rusakov, Phys. Lett. **B303** (1993) 95.

[10] M. R. Douglas and V. A. Kazakov, Phys. Lett. **B319** (1993) 219.

[11] B. Rusakov and S. Yankielowicz, Phys. Lett. **B339** (1994) 258.

[12] V. S. Popov and A. M. Perelomov, Soviet Math. Dokl. **8** (1967) 712.

[13] E. Brezin, C. Itzykson, G. Parisi and J. B. Zuber, Commun. Math. Phys. **59** (1978) 35.

[14] D. Gakhov, "Boundary problems", Russian edition (Nauka, 1975);
A. C. Pipkin, "A course on integral equations" (Springer, Berlin, 1991).

[15] J. A. Minahan and A. P. Polychronakos, Nucl. Phys. **B422** (1994) 172.

[16] A. Aghamohammadi, M. Alimohammadi and M. Khorrami, "Uniqueness of the minimum of the free energy of the 2D Yang-Mills theory at large N", hep-th/9707080